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Instabilities of Electroweak Strings

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Abstract

We investigate the instabilities of low winding number electroweak strings using standard numerical techniques of linear algebra. For strings of unit winding we are able to confirm and extend existing calculations of the unstable region in the $(m_H/m_W, \sin^2\theta_W)$ plane. For strings of higher winding number we map the unstable regions for the various decay modes.

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It has been known for some time that certain field theory models with spontaneous symmetry breaking contain toplogical defects, such as domain walls, strings and monpoles. These defects are stable because their field configurations wrap around the vacuum manifold, and to unwind them requires lifting the Higgs field to its vacuum value in the unbroken phase,throughout an infinite volume of space. There is therefore an infinite energy barrier separating these defect solutions from the vacuum.

Recently, defect solutions have been found in theories where they are not topologically stable [1, 2, 3]. These solutions may be a maximum or a local minimum of the energy, depending upon the dynamics of the model in question. Such 'embedded defects' have been found in the electroweak model: $SU(2)_L \times U(1)_Y \to U(1)_{em}$. This contains two embedded $U(1) \to I$ symmetry breakings which can both give rise to Nielsen-Olesen vortex solutions [3]. These solutions are called τ- and Z-strings depending upon how one chooses the embedding. τ-strings are expected to be unstable for all values of the parameters [4] and so we will only consider Z-strings.

The stability of Z-strings for a range of parameters was addressed in James *et al* [5] where it was shown that for physical values of the parameters the $N = 1$ string is unstable. The stability analysis did not extend down to small values of the Higgs self coupling because of numerical problems caused by the widely different scales of the two cores of the string. The method adopted for fixing the gauge, locating the unstable mode and computing the eigenvalues was not wholly transparent either.

In Perkins [6] and Achúcarro *et al* [7] instabilities due to W-condensation were investigated, and in Achúccaro *et al* it was shown that a W-condensed string of winding number N was equivalent to a string of winding number $N-1$. In addition to this decay mode Achúccaro *et al* also showed that there is another distinct physical decay mode for large N strings, where the upper component of the Higgs field acquires a non-zero value at the string core. This decay mode should result in the complete unwinding of the string.

MacDowell and Törnkvist [8] showed that for some, unspecified, regions of the parameter space a string of winding number N is unstable to the formation of W-fields with angular momentum m such that $-2N < m < 0$.

In this letter we seek to complete the stability line of James *et al*, investigate the two decay modes of Achúcarro *et al* and investigate the $2N-1$ decay modes of MacDowell and Törnkvist. We are careful to fix the gauge at the outset, and we use more usual eigenvalue methods than were adopted in James *et al*. We do not, however, follow Achúcarro *et al* in tracking the full non-linear evolution of the fields.

We consider the bosonic sector of the Weinberg-Salam model with the lagrangian

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \lambda(\phi^\dagger\phi - \eta^2/2)^2,$$

where

$$\begin{aligned} D_\mu &= (\partial_\mu - ig\sigma^a W^a_\mu/2 - ig' B_\mu/2)\phi \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned}$$

$$G^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu.$$

σ^a are the Pauli spin matrices, W^a_μ are the $SU(2)$ gauge fields, B_μ are the $U(1)$ gauge fields and ϕ is the Higgs doublet. The $SU(2)_L \times U(1)_Y$ symmetry of this lagrangian is broken to the $U(1)_{em}$ symmetry of electromagnetism by the Higgs field acquiring a vacuum expectation value $\phi^\dagger = (0, \eta/\sqrt{2})$. With the standard field basis

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \qquad A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$
$$W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu) \qquad W^-_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu)$$

the boson masses generated by the Higgs mechanism are

$$M_W = \frac{1}{2}g\eta, \qquad M_Z = \frac{1}{2}g_z\eta, \qquad M_H = \sqrt{2\lambda\eta^2}.$$

The Weinberg angle θ_W is given by $\tan\theta_W = g'/g$, and $g_z = \sqrt{g^2 + g'^2}$.

Consider time independent field configurations with $\phi_u = W^+_\mu = W^-_\mu = A_\mu = 0$ and $Z_0 = 0$, then the energy functional is given by

$$E = \int d^3x \left(\frac{1}{4}Z_{ij}Z_{ij} + |(\nabla_i + \frac{ig_z}{2}Z_i)\phi_d|^2 + \lambda(|\phi_d|^2 - \eta^2/2)^2\right),$$

where

$$Z_{ij} = \nabla_i Z_j - \nabla_j Z_i.$$

This is the same as for the Abelian-Higgs model and so there is a Nielsen-Olesen string configuration given by

$$\phi_d = \frac{\eta}{\sqrt{2}}f(r)e^{iN\theta}, \qquad Z_r = 0, \qquad Z_\theta = -\frac{2N}{g_z r}a(r),$$

which extremises the energy. The functions $f(r)$ and $a(r)$ are identical to those found by Nielsen and Olesen [9].

Consider a general perturbation to the solution of the form

$$\begin{aligned}
\phi_u &= \epsilon\delta\phi_u e^{-i\omega t} & \phi^*_u &= \epsilon\delta\phi^*_u e^{i\omega t}\\
\phi_d &= \phi^c_d + \epsilon\delta\phi_d e^{-i\omega t} & \phi^*_d &= \phi^{c*}_d + \epsilon\delta\phi^*_d e^{i\omega t}\\
W^+_\mu &= \epsilon\delta W^+_\mu e^{-i\omega t} & W^-_\mu &= \epsilon\delta W^-_\mu e^{-i\omega t}\\
Z_\mu &= Z^c_\mu + \epsilon\delta Z_\mu e^{-i\omega t} & A_\mu &= \epsilon\delta A_\mu e^{-i\omega t}.
\end{aligned}$$

This gives corrections to the action

$$S = S(\phi^c_d, Z^c_\mu) + \epsilon^2\frac{1}{2}\int d^4x \delta\chi^\dagger\mathcal{D}\delta\chi + O(\epsilon^3),$$

where $\mathcal{D}$ is the perturbation operator and

$$\delta\chi^\dagger = (\delta\phi_u e^{-i\omega t}, \delta\phi_u^* e^{i\omega t}, \delta W_\mu^+ e^{-i\omega t}, \delta W_\mu^- e^{-i\omega t}, \delta\phi_d e^{-i\omega t}, \delta\phi_d^* e^{i\omega t}, \delta Z_\mu e^{-i\omega t}, \delta A_\mu e^{-i\omega t}).$$

The equations of motion for the perturbations $\mathcal{D}\delta\chi = 0$, could in principle be coupled equations in twenty fields. For the string background however, they separate into four sets of coupled equations. The first two are of the form

$$\mathcal{D}_1 \begin{pmatrix} \delta\phi_u \\ \delta W_\mu^+ \end{pmatrix} = 0, \qquad \mathcal{D}_2 \begin{pmatrix} \delta\phi_u^* \\ \delta W_\mu^- \end{pmatrix} = 0, \tag{1}$$

which, since the string solution is invariant under charge conjugation, are just charge conjugates, so we need only consider the first of these. The third is of the form

$$\mathcal{D}_3 \begin{pmatrix} \delta\phi_d \\ \delta\phi_d^* \\ \delta Z_\mu \end{pmatrix} = 0,$$

which contains only the fields of the Nielsen-Olesen vortex, and was addressed in [10]. The fourth equation is

$$\mathcal{D}_4 \delta A_\mu = (\eta^{\mu\nu}\partial^2 - \partial^\nu\partial^\mu)\delta A_\mu = 0,$$

which is just the usual wave equation of electromagnetism. The last two equations do not contain any modes which can decrease the winding number, and so we must look at the solutions of (1) above for decay modes. As mentioned in [10] these equations contain linear derivative terms and gauge degrees of freedom, both of which can be removed by choosing the background gauge conditions

$$\begin{aligned} F_1(W_\mu^+) &= \partial^\mu W_\mu^+ - ig\cos\theta_W Z_c^\mu W_\mu^+ - \frac{ig}{\sqrt{2}}\phi_d^{c*}\phi_u = 0 && (2)\\ F_2(W_\mu^-) &= \partial^\mu W_\mu^- + ig\cos\theta_W Z_c^\mu W_\mu^- + \frac{ig}{\sqrt{2}}\phi_d^c\phi_u^* = 0 && (3)\\ F_3(Z_\mu) &= \partial^\mu Z_\mu - \frac{ig_z}{2}(\phi_d^*\phi_d^c - \phi_d^{c*}\phi_d) = 0 && (4)\\ F_4(A_\mu) &= \partial^\mu A_\mu = 0 && (5) \end{aligned}$$

which are imposed by adding the gauge fixing terms

$$\mathcal{L}_{\mathcal{GF}} = \frac{1}{2}\sum_{i=1}^{4}|F_i|^2$$

to the lagrangian. This enables us to separate out the linear time derivatives and to set up gauge-fixed eigenvalue equations, which for equation (1) above are

$$M^{GF}\begin{pmatrix} \delta\phi_u \\ \delta W_i^+ \end{pmatrix} = \omega^2 \begin{pmatrix} \delta\phi_u \\ \delta W_i^+ \end{pmatrix} \tag{6}$$

and

$$(-\nabla^2 + 2ig\cos\theta_W Z_k^c \nabla_k + g^2\cos^2\theta_W Z_k^c Z_k^c + \frac{g^2}{2}|\phi_d^c|^2)\begin{pmatrix}\delta W_0^+ \\ \delta W_z^+\end{pmatrix} = \omega^2 \begin{pmatrix}\delta W_0^+ \\ \delta W_z^+\end{pmatrix}, \quad (7)$$

where $i = 1, 2$, and the gauge fixed perturbation operator is given by

$$M^{GF} = \begin{pmatrix} D_1 & X_i \\ X_j^* & D_{2_{ji}} \end{pmatrix}$$

where

$$\begin{aligned}
D_1 &= -\nabla^2 + ig_z\cos 2\theta_W Z_k^c\nabla_k + \frac{g_z^2}{4}\cos^2 2\theta_W Z_k^c Z_k^c + 2\lambda(|\phi_d^c|^2 - \eta^2/2) + \frac{g^2}{2}|\phi_d^c|^2 \\
D_{2_{ji}} &= \delta_{ji}(-\nabla^2 + 2ig\cos\theta_W Z_k^c\nabla_k + g^2\cos^2\theta_W Z_k^c Z_k^c + \frac{g^2}{2}|\phi_d^c|^2) + 2ig\cos\theta_W Z_{ji}^c \\
X_i &= ig\sqrt{2}(\nabla_i + \frac{ig_z}{2}Z_i^c)\phi_d^c \\
X_j^* &= -ig\sqrt{2}(\nabla_j - \frac{ig_z}{2}Z_j^c)\phi_d^{c*}.
\end{aligned}$$

The δW_0^+ and δW_z^+ perturbations decouple because the background string solution is independent of t and z.

The gauge condition used means that the gauge fixing terms must be accompanied by Fadeev-Popov ghost terms. This results in an accompanying eigenvalue equation for the ghost field Λ^+

$$(-\nabla^2 + 2ig\cos\theta_W Z_k^c\nabla_k + g^2\cos^2\theta_W Z_k^c Z_k^c + \frac{g^2}{2}|\phi_d^c|^2)\Lambda^+ = \omega^2\Lambda^+. \quad (8)$$

This is the same equation as (7) and so Λ^+, δW_0^+ and δW_z^+ all have the same eigenvalue spectra. Of the total of 5 eigenmodes in (6), (7) and (8), only three should be physical, corresponding to the three spin states of the massive W boson. This is ensured by the ghosts canceling one linear combination of δW_0^+ and δW_z^+, and one of the eigenmodes of M_{GF}.

It may seem that the gauge choice (2)-(5) introduces unnecesary complications and a difficulty in identifying the physical modes. However, we are looking for decay modes where $\omega^2 < 0$ and the eigenvalues of (8) are positive, so there is no difficulty in identifying the field configurations of the decay modes. It is also worth pointing out that (2)-(5) results in a considerable simplication in the resulting eigenvalue equations (as opposed to the temporal gauge for instance).

We expand the scalar and gauge fields in total angular momentum states. For the gauge fields they are

$$W_\uparrow^+ = \frac{e^{-i\theta}}{\sqrt{2}}(W_r^+ - \frac{i}{r}W_\theta^+), \qquad \text{and} \qquad W_\downarrow^+ = \frac{e^{i\theta}}{\sqrt{2}}(W_r^+ + \frac{i}{r}W_\theta^+).$$

The total angular momentum operator for the gauge fields is $J_z = L_z + S_z$ where

$$L_z = -i\frac{d}{d\theta}, \qquad \text{and} \qquad (S_z W^+)_j = -i\epsilon_{3jk} W_k^+,$$

with $(S_z W^+)_\uparrow = +W_\uparrow^+$, and $(S_z W^+)_\downarrow = -W_\downarrow^+$. So the suffices $(\uparrow)$ and $(\downarrow)$ identify gauge fields with spin up and spin down respectively. The background string solution in total angular momentum states is

$$\phi_d^c = \frac{\eta}{\sqrt{2}} f(r) e^{iN\theta}, \qquad Z_\uparrow^c = \frac{i\sqrt{2}N}{g_z r} a(r) e^{-i\theta}, \qquad Z_\downarrow^c = \frac{-i\sqrt{2}N}{g_z r} a(r) e^{i\theta},$$

and the perturbations are

$$\delta\phi_u = \sum_{m'} s_{m'} e^{im'\theta}, \qquad \delta W_\uparrow^+ = \sum_m -i w_m e^{i(m-1)\theta}, \qquad \delta W_\downarrow^+ = \sum_m i w_{-m}^* e^{i(m+1)\theta},$$

with $N + m = m'$. It is useful to rescale

$$\phi = \frac{\eta}{\sqrt{2}}\phi, \quad Z_i = \frac{\eta}{\sqrt{2}} Z_i, \quad W_i^+ = \frac{\eta}{\sqrt{2}} W_i^+, \quad W_i^- = \frac{\eta}{\sqrt{2}} W_i^-, \quad r = \frac{2\sqrt{2}}{g_z \eta}\rho,$$

to give dimensionless variables. The eigenvalues are now in units $g_z^2\eta^2/8$ and the magnetic field strength is in units $g_z\eta^2/4 = m_Z^2/g_z$. Substituting in the above gives the eigenvalue equations

$$\begin{pmatrix} D_1 & A & B \\ A & D_2 & 0 \\ B & 0 & D_3 \end{pmatrix} \begin{pmatrix} s_{m'} \\ w_m \\ w_{-m}^* \end{pmatrix} = \omega^2 \begin{pmatrix} s_{m'} \\ w_m \\ w_{-m}^* \end{pmatrix} \tag{9}$$

where

$$\begin{aligned}
D_1 &= -\nabla_\rho^2 + \frac{(m' + aN\cos^2\theta_W)^2}{\rho^2} + \beta(f^2 - 1) + 2f^2\cos^2\theta_W \\
D_2 &= -\nabla_\rho^2 + \frac{((m-1) + 2aN\cos^2\theta_W)^2}{\rho^2} + 2f^2\cos^2\theta_W + 4\cos^2\theta_W \frac{N}{\rho}\frac{da}{d\rho} \\
D_3 &= -\nabla_\rho^2 + \frac{((m+1) + 2aN\cos^2\theta_W)^2}{\rho^2} + 2f^2\cos^2\theta_W - 4\cos^2\theta_W \frac{N}{\rho}\frac{da}{d\rho} \\
A &= 2\left(\nabla_\rho f - \frac{Nf}{\rho}(1-a)\right) \\
B &= -2\left(\nabla_\rho f + \frac{Nf}{\rho}(1-a)\right)
\end{aligned}$$

and $\beta = 8\lambda/g_z^2$.

If we resolve $s_{m'}, w_m, w_{-m}^*$ into real and imaginary parts, the complex eigenvalue problem seperates into two eigenvalue problems with explicitly real fields

$$\begin{pmatrix} D_1 & A & B \\ A & D_2 & 0 \\ B & 0 & D_3 \end{pmatrix} \begin{pmatrix} s^r_{m'} \\ w^r_m \\ w^r_{-m} \end{pmatrix} = \omega^2 \begin{pmatrix} s^r_{m'} \\ w^r_m \\ w^r_{-m} \end{pmatrix}$$

and

$$\begin{pmatrix} D_1 & A & B \\ A & D_2 & 0 \\ B & 0 & D_3 \end{pmatrix} \begin{pmatrix} s^i_{m'} \\ w^i_m \\ -w^i_{-m} \end{pmatrix} = \omega^2 \begin{pmatrix} s^i_{m'} \\ w^i_m \\ -w^i_{-m} \end{pmatrix}$$

where D_γ, A and B are given above. The profiles for the Nielsen-Olesen vortex were solved for with a relaxation method on the energy functional, and then substituted into the eigenvalue equations above. This eigenvalue problem was solved for a range of β and θ_W with the boundary conditions $s_{m'}, w_m, w_{-m} \to 0$ as $\rho \to \infty$ by the standard matrix packages incorporated into MATLAB.

For $1 > \beta > 0.04$ a linear discretisation of ρ was used. For $\beta < 0.04$ the scales of the Higgs core and the gauge field core are too different to use this linear descretisation so we followed Yaffe [11] in using the convenient transformation

$$\xi = \frac{1}{\ln(m_H/m_Z)} \ln\left(\frac{1 + m_H r}{1 + m_Z r}\right).$$

This maps the line $0 \le r < \infty$ to $0 \le \xi < 1$. Linear discretisation of ξ results in more points in the core region and is better suited to studying very small and very large values of $\beta = m_H/m_Z$. With this map we were able to look at β down to 1×10^{-4}.

The resulting stability lines are shown in the Figure. All the lines for the various decay modes separate the region on the left hand side where the string is unstable, from the region on the right hand side where the string is stable with respect to a particular decay mode.

The two decay modes of Achúcarro *et al* correspond to $m' = 0$ and $m = -1$. For the $m' = 0$ mode ϕ_u acquires a non-zero value at the core of the string, and then there is nothing to stop the string from completely unwinding and decaying to the vacuum. For the $m = -1$ mode $W^+_\downarrow$ acquires a non-zero value at the core of the string, where its magnetic moment is aligned with the magnetic field of the string. The term giving rise to this W-condensation is $-4\cos^2\theta_W \delta W^+_\downarrow N a'/\rho$, which is of the generic form $-\mathbf{m}.\mathbf{B}$ for the interaction between a magnetic moment $\mathbf{m}$ and a magnetic field $\mathbf{B}$. This term comes from the non-abelian nature of the field tensor. Achúcarro *et al* showed that a W-condensed string of winding number N has the same energy as a string of winding $N-1$, and so the $m = -1$ mode corresponds to a unit unwinding. It is this $m = -1$ mode which is refered to as W-condensation in [6] and [7]. In the background gauge, however, far from the string ϕ_u can readily be indentified as the longitudinal W gauge field, and so all the modes we have found can be called W-condensation.

For the $N=1$ string the relation $N+m=m'$ gives that the $m'=0$ and $m=-1$ modes are one and the same with both ϕ_u and $W_\downarrow^+$ acquring non-zero values at the core of the string. The upper part of the stability line in the Figure agrees with that found by James *et al.* This is different from the stability line in [6] ($\sqrt{\beta}=4\cos\theta_W$) which was found analytically by using fairly crude approximations for the string solution and for the profiles of the W-fields. A large part of this difference could be that the decay mode of the $N=1$ string has non-trivial field configurations for the W-fields and for ϕ_u as well, whereas the analysis in [6] contained no ϕ_u field.

For the $N=2$ string as well as the two decay modes mentioned above there is a third decay mode for which $m'=-1$, $m=-3$. We conjecture that this decay mode corresponds to the $N=2$ string decaying to a string of winding number $N=-1$. Since the Z-strings are non-topological and an $N=-1$ string has the same energy as an $N=1$ string, such a decay is to be expected, albeit with a larger energy barrier than the decay to an $N=1$ string. This later point is consistent with the $m'=-1$ decay mode having a much larger region of stability in the parameter space than the decay mode to an $N=1$ string.

Generalising this to strings of winding number N, means that we would expect there to be $2N-1$ decay modes corresponding to the string decaying to strings of winding number $N-1,\ldots,-N+1$. Furthermore we would expect that the stability lines for these decays to be such that those for larger units of unwinding would have greater regions of stability, due to the increasing energy barrier for such decays.

The stability lines for the $N=3$ string, appear to be consistent with this interpretation (as are those for $N=1$ and $N=2$ strings). It is also consistent with the result of MacDowell and Törnkvist that strings of winding number N have regions in the parameter space for which they are unstable to the formation of W-fields with angular momentum m such that $-2N<m<0$.

For the strings we studied ($N=1,2,3$), all have a stability line close to the $\sqrt{\beta}$ axis for the mode with $m=-2N$. For all the strings this line is in approximately the same place ($\sin^2\theta_W\simeq 0.02$) and approaches the $\sqrt{\beta}$ axis as the number of points in the lattice is increased. We believe that in the continuum limit this $m=-2N$ mode will have zero eigenvalue along the line $\sin^2\theta_W=0$, and the deviation of the line from this value for finite lattices (100 points) is an indication of the error.

For the complex conjugate problem with the fields ϕ_u^* and W^-,the stability lines are of course the same. The 'W-condensation' mode is where $W_\uparrow^-$ is non-zero at the core of the string, so the magnetic moment is still aligned with the magnetic field of the string.

It must be noted that all the modes involve non-trivial field configurations for ϕ_u,$W_\uparrow^+$ and $W_\downarrow^+$.

Finally, we consider the large N limit. If we assume that the ϕ_u and W perturbations drop to zero whilst inside the core, we need only consider the small ρ behaviour of the string solution, which is $f\rightarrow f_N\rho^N$, $a\rightarrow\rho^2/2$ as $\rho\rightarrow 0$, where f_N is a constant.

Substituting this into the perturbation equation (9) gives

$$\begin{aligned}
D_1 &= -\nabla_\rho^2 + \frac{(m' + \rho^2 N \cos 2\theta_W/2)^2}{\rho^2} + \beta(f_N^2\rho^{2N} - 1) + 2f_N^2\rho^{2N}\cos^2\theta_W \\
D_2 &= -\nabla_\rho^2 + \frac{((m-1) + \rho^2 N \cos^2\theta_W)^2}{\rho^2} + 2f_N^2\rho^{2N}\cos^2\theta_W + 4N\cos^2\theta_W \\
D_3 &= -\nabla_\rho^2 + \frac{((m+1) + \rho^2 N \cos^2\theta_W)^2}{\rho^2} + 2f_N^2\rho^{2N}\cos^2\theta_W - 4N\cos^2\theta_W \\
A &= f_N N \rho^{N+1} \\
B &= f_N N \rho^{N+1} - 4f_N N \rho^{N-1}.
\end{aligned}$$

So for large N, the perturbations in ϕ_u and the W fields decouple. Considering terms up to ρ^2 it was shown in [7] that the equation for ϕ_u for $m' = 0$ has a stability line given by $\sqrt{\beta} = |1 - 2\sin^2\theta_W|$. The Figure seems to indicate that the $m' = 0$ line is approaching this limit but there is no stability region in the bottom left of the plot, which is not altogether suprising since $N = 3$ is not large N.

Lastly, we present an explanation for the movement of the $m = -1$ stabilitiy line towards the line $\sin^2\theta_W = 1$ for increasing winding number. Consider the equation for $W_\downarrow^+$ with $m = -1$ for large N, with the assumption that the perturbation drops rapidly to zero within the core, so that we need keep terms only up to $O(\rho^2)$:

$$\left(-\frac{d^2}{d\rho^2} - \frac{1}{\rho}\frac{d}{d\rho} + N^2\rho^2\cos^4\theta_W - 4N\cos^2\theta_W\right) w_1^r = \omega^2 w_1^r.$$

This has a solution $w_1^r = c\exp(-N\cos^2\theta_W\rho^2/2)$ for which $\omega^2 = -2N\cos^2\theta_W$, and so large N strings are unstable to 'W-condensation' for all values of the parameters. This is consistent with the stability regions of the $m = -1$ modes displayed in the Figure.

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Figure captions

Figure 1: The stability lines for a) the $N = 1$ string (solid line), b) the $N = 2$ string (dashed line) and c) the $N = 3$ string (dotted line). The angular momentum m of the modes are (reading from right to left) a) $m = -1$, b) $m = -1, -2, -3$, c) $m = -1, -2, -3, -4, -5$.

Figure 1.